Trinity College

WA Exams Practice Paper D, 2015

Question/Answer Booklet

MATHEMATICS SPECIALIST UNIT 1

Section Two: Calculator-assumed

SOLUTIONS

Student Number: In figures

In words ______________________________

Your name ______________________________

Time allowed for this section

Reading time before commencing work: ten minutes
Working time for this section: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	12	12	100	98	65
			Total	150	100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2015*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

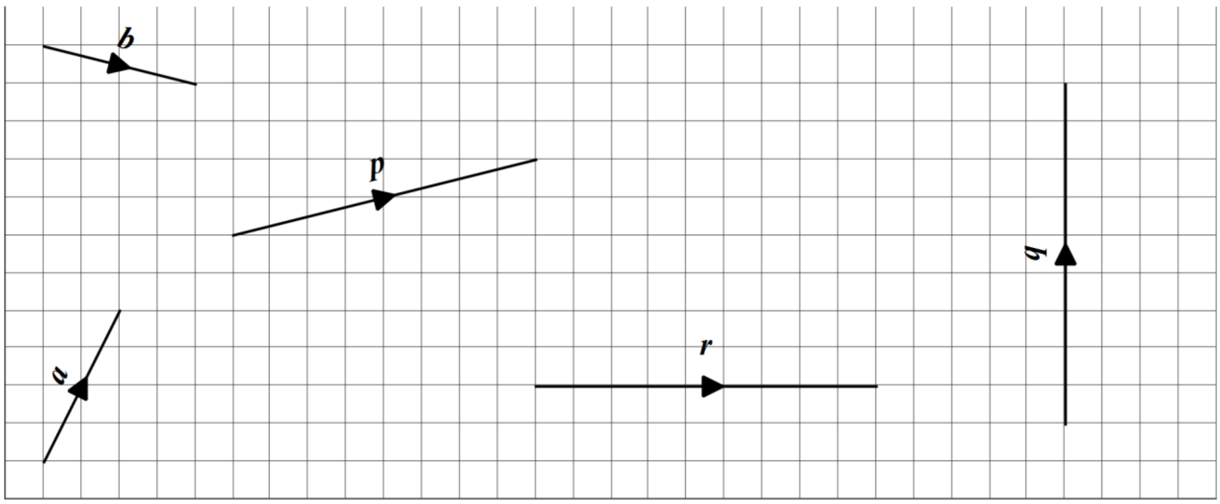
Section Two: Calculator-assumed **(98 Marks)**

This section has **twelve (12)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 100 minutes.

Question 9 **(9 marks)**

Vectors **a** and **b** are as shown on the grid below.

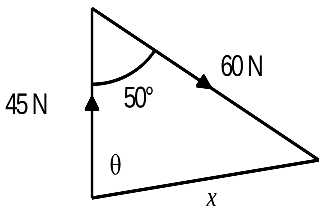


(a) On the grid above, sketch and label the vectors **p**, **q** and **r** where

$$\mathbf{p} = \mathbf{a} + 2\mathbf{b}$$
$$\mathbf{q} = 2\mathbf{a} - \mathbf{b}$$
$$\mathbf{r} = -2\mathbf{b} - 0.5\mathbf{a}$$

(4 marks)

(b) Two vectors have magnitudes of 45 N and 60 N and the angle between their directions is 130°. Sketch a diagram to show their sum and use trigonometry to calculate the magnitude of the resultant and the angle it makes with the smaller vector. (5 marks)



$$x^2 = 45^2 + 60^2 - 2 \times 45 \times 60 \cos 50$$
$$x = 46.41 \text{ N}$$

$$\frac{\sin\theta}{60} = \frac{\sin 50}{46.41}$$
$$\theta = 82.0°$$

Question 10 **(8 marks)**

(a) Of 56 people interviewed on their means of transportation to work over the past month, 38 had used a bus, 19 had cycled and 24 had used a car. On further analysis it was found that 12 had taken the car and also cycled, 16 had used both bus and car, 4 had used all three means and 5 said they did not use any of them.

(i) Explain why it follows from the above information that 51 people either caught a bus, cycled or used a car to get to work. (1 mark)

$$n(U) = n(A \cup B \cup C) + n(\overline{A \cup B \cup C})$$
$$56 = n(A \cup B \cup C) + 5$$
$$n(A \cup B \cup C) = 51$$

(ii) Determine how many people used both bus and cycle to get to work over the time period. (3 marks)

$$51 = 38 + 19 + 24 - 12 - 16 - x + 4$$

$$x = 6$$

(b) A box contains 630 coloured cubes in equal numbers of red, blue, green, orange, black and yellow. How many cubes must be selected from the box to ensure that at least five of one colour are in the selection? Justify your answer. (2 marks)

Possible to take a maximum of $4 \times 6 = 24$ cubes with no more than four of each colour. Taking one more cube ensures at least five of one colour, so must select **25** cubes.

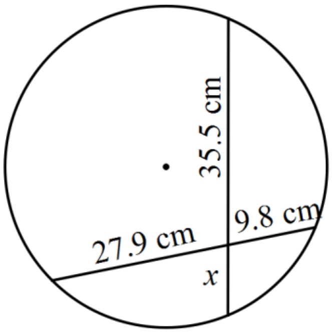
(c) 47 students answered a spelling quiz. One student made seven errors, with the rest making fewer errors than this student. Prove that at least seven of the students made an equal number of errors. (2 marks)

Using the pigeonhole principle to model this situation, we have seven holes (corresponding to 0 to 6 errors) to be filled with the other 46 students (pigeons).

We can evenly distribute 6 students in each pigeonhole, but we are still left with 4 to place.
So at least one of the pigeonholes must contain seven or more.

Question 11 **(8 marks)**

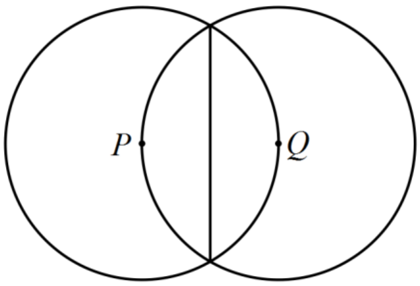
(a) Evaluate the length of x, correct to one decimal place. (2 marks)



$$\frac{x}{27.9} = \frac{9.8}{35.5}$$

$$x = 7.702$$
≈ 7.7 cm (1dp)

(b) Two circles with the same radius and centres P and Q intersect as shown. The length of the common chord joining the points of intersection is 20 cm. Determine the exact radius of the circles. (3 marks)



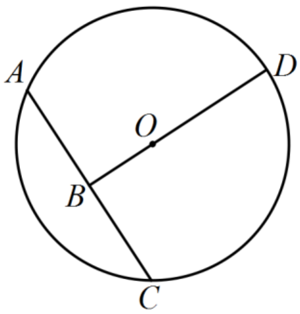
Let x be the half radius.

$$10^2 + x^2 = (2x)^2$$

$$x = \frac{10\sqrt{3}}{3}$$

$$r = \frac{20\sqrt{3}}{3}$$ cm

(c) In the circle shown below with centre O and radius 30 cm, the lengths AB and CB are equal, as are the lengths AC and BD. Determine the length OB. (3 marks)



Let $x = OB$.

$$AB = \sqrt{30^2 - x^2}$$

$$2\sqrt{30^2 - x^2} = 30 + x$$

$$x = -30, x = 18$$

$OB = 18$ cm

Question 12 **(7 marks)**

(a) Use the scalar product to show that

(i) the vectors (3, -2) and (4, 6) are perpendicular. (1 mark)

$$3\times 4+(-2)\times 6=0$$

(ii) the vectors $\begin{bmatrix}-1\\3\end{bmatrix}$ and $\begin{bmatrix}0.5\\-1.5\end{bmatrix}$ are parallel. (2 marks)

$$\left\|\begin{bmatrix}-1\\3\end{bmatrix}\right\|\times\left\|\begin{bmatrix}0.5\\-1.5\end{bmatrix}\right\|=\sqrt{10}\times\sqrt{\tfrac{10}{4}}=\sqrt{\tfrac{100}{4}}=5$$

$$-1\times 0.5+3\times -1.5=-5$$

Hence parallel (-ve indicates opposite direction)

(b) Determine

(i) the vector projection of a force of 46N on bearing 015º onto a force of 87N on a bearing 075º. (2 marks)

$$46\cos(75-15)=23$$

Projection is force of 23N on bearing 075º.

(ii) the vector projection of $3\mathbf{i}-\mathbf{j}$ onto $-4\mathbf{i}+3\mathbf{j}$. (2 marks)

$$\frac{3\times(-4)+(-1)\times 3}{(-4)\times(-4)+3\times 3}(-4\mathbf{i}+3\mathbf{j})$$

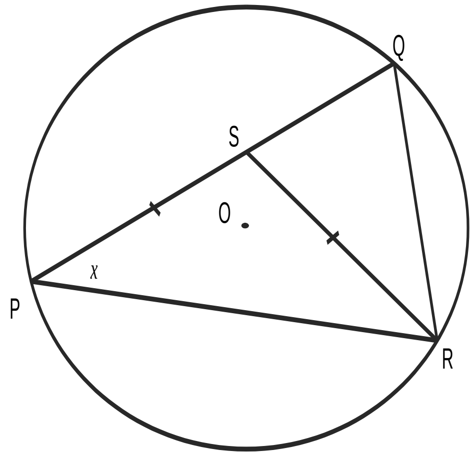
$$=-\tfrac{15}{25}(-4\mathbf{i}+3\mathbf{j})$$

$$=\tfrac{3}{5}(4\mathbf{i}-3\mathbf{j})$$

Question 13 **(6 marks)**

The three vertices of $\triangle PQR$ lie on a circle with centre O.

Point S lies on PQ such that the lengths PS and RS are equal and $\angle QPR = x$.



(a) Explain why $\angle QOR = 2x$. (1 mark)

> The angle standing on a common chord (QR) at the centre of circle (QOR) is twice angle on circumference (QPR).

(b) Prove that $QROS$ is a cyclic quadrilateral. (3 marks)

> $\angle PRS = x$ ($\triangle PRS$ is isosceles).
>
> $\angle QSR = 2x$ (external angle of triangle ($\triangle PRS$) is sum of opposite interior angles of triangle ($\angle SPR$ and $\angle SRP$)).
>
> Note that $\angle QSR = \angle QOR = 2x$.
>
> Hence $QROS$ is a cyclic quadrilateral. (Angles standing on a common chord (QR) on the circumference of a circle are equal).

(c) Calculate the area of $\triangle QRS$ given the area of $\triangle PRS$ is 30 cm^2 and the ratio of lengths $QS : QP = 2 : 5$. (2 marks)

> Ratio of areas of $\triangle QRS : \triangle PRS = 2 : 3$
>
> Area of $\triangle QRS = \frac{2}{3} \times 30 = 20$ cm^2

Question 14 **(9 marks)**

(a) The letters of the word CLASSICS are to be arranged in a row.

(i) Determine the number of ways in which this can be done. (2 marks)

$$\frac{8!}{3!2!} = 3360$$

(ii) How many of the arrangements end with a letter other than S? (2 marks)

End with S: $\frac{3 \times 7!}{3!2!} = 1260$

Don't end with S: $3360 - 1260 = 2100$

(b) Even numbers are to be formed using some, or all, of the digits 5, 6, 7, 8 and 9.

(i) How many even numbers can be formed in this way, if repetition of digits is not allowed? (3 marks)

1-digit: 2
2-digit: $4 \times 2 = 8$
3-digit: $4 \times 3 \times 2 = 24$
4-digit: $4 \times 3 \times 2 \times 2 = 48$
5-digit: $4 \times 3 \times 2 \times 1 \times 2 = 48$

Total = 130 even numbers

(ii) What fraction of the even numbers in (i) start with a 9? (2 marks)

2-digit: $1 \times 2 = 2$
3-digit: $1 \times 3 \times 2 = 6$
4-digit: $1 \times 3 \times 2 \times 2 = 12$
5-digit: $1 \times 3 \times 2 \times 1 \times 2 = 12$

Fraction $= \frac{32}{130}$

Question 15 **(8 marks)**

At 8 am one morning two ships, A and B, have position vectors **r** km and velocity vectors **v** km/h as follows:

$$\mathbf{r}_A = 80\mathbf{i} - 20\mathbf{j} \qquad \mathbf{v}_A = -5\mathbf{i} + 14\mathbf{j} \qquad \mathbf{r}_B = -20\mathbf{i} + 30\mathbf{j} \qquad \mathbf{v}_B = 15\mathbf{i} + 6\mathbf{j}$$

(a) Relative to ship B,

(i) determine the position of ship A at 8 am. (1 mark)

$$\begin{bmatrix} 80 \\ -20 \end{bmatrix} - \begin{bmatrix} -20 \\ 30 \end{bmatrix} = \begin{bmatrix} 100 \\ -50 \end{bmatrix}$$

(ii) determine the velocity of ship A. (1 mark)

$$\begin{bmatrix} -5 \\ 14 \end{bmatrix} - \begin{bmatrix} 15 \\ 6 \end{bmatrix} = \begin{bmatrix} -20 \\ 8 \end{bmatrix}$$

(iii) Explain why it is not necessary to know the time to obtain your answer in (ii). (1 mark)

In this situation where both ships have constant velocities, relative velocity is independent of time.

(b) Determine the distance between ship A and ship B later the same day at

(i) 2 pm. (2 marks)

$$t = 6, \left\| \begin{bmatrix} -20 \\ -2 \end{bmatrix} \right\| = 2\sqrt{101} \approx 20.1 \text{ km}$$

(ii) 6 pm. (1 mark)

$$10\sqrt{109} \approx 104 \text{ km}$$

(c) Is the distance between the two ships always increasing after 8 am on this day? Justify your answer. (2 marks)

No.

Possible justification: At 8 am distance apart was just under 112 km, so must have decreased at some time since only 20 km apart at 2 pm.

Question 16 **(7 marks)**

A retailer has discounted the prices on 32 different music CD's, with 11 of them priced at $8 each and the rest on sale at $12 each.

(a) If a customer spent exactly $20, how many different combinations of CD's could they have bought? (1 mark)

$11 \times 21 = 231$

(b) If a customer spent exactly $24, how many different combinations of CD's could they have bought? (2 marks)

Choose 3 of the $8 CD's

${}^{11}C_3 = 165$

or choose 2 of the $12 CD's

${}^{21}C_2 = 210$

to get a total of 375 combinations

(c) If a customer decides to pick at least one CD and to spend no more than $24, how many different combinations of CD's could they buy? (4 marks)

Choose 1 CD: 1 of the $8 or 1 of the $12

${}^{11}C_1 + {}^{21}C_1 = 11 + 21 = 32$

Choose 2 CD's: 2 of the $8 or 2 of the $12 or 1 of each

${}^{11}C_2 + {}^{21}C_2 + {}^{11}C_1 \times {}^{21}C_1 = 55 + 210 + 11 \times 21 = 496$

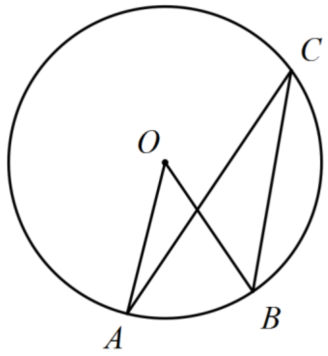
Choose 3 CD's: 3 of the $8

${}^{11}C_3 = 165$

to get a total of 32+496+165=693 combinations

Question 17 **(7 marks)**

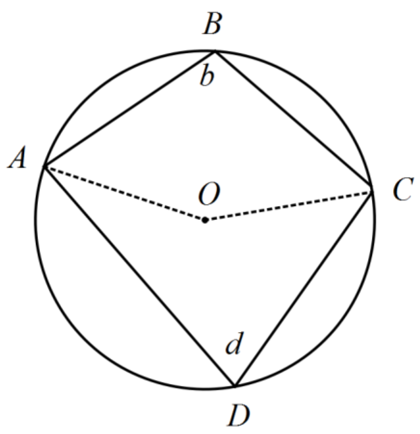
(a) In the diagram below, where points A, B and C lie on the circumference of a circle with centre O, $\angle ACB = 32°$ and $\angle OAC = 18°$. Determine the size of $\angle OBC$. (2 marks)



$\angle AOB = 2 \times 32$
$= 64$
$\angle OPA = 180 - 64 - 18$
$= 98$
$= \angle BPC$
$\angle OBC = 180 - 98 - 32$
$= 50°$

(P is intersection of OB and AC)

(b) Use proof by contradiction to prove that the opposite angles of a cyclic quadrilateral are supplementary. (5 marks)



Let A, B, C and D be four points lying on a circle with centre O, and let $\angle ABC = b$ and $\angle ADC = d$ such that $b + d \neq 180°$.

$\angle AOC = 2d$ (obtuse) and $\angle AOC = 2b$ (reflex) since the angle at centre is twice angle on circumference.

So sum of angles at point O is $2b + 2d$ and from our original assumption $b + d \neq 180° \Rightarrow 2b + 2d \neq 360$.

But this contradicts the fact that the angle at a point is always 360º and so our original assumption must be wrong and hence $b + d = 180°$.

Question 18 **(9 marks)**

The points A, B and C have position vectors $\mathbf{a} = 3\mathbf{i} + 4\mathbf{j}$, $\mathbf{b} = 14\mathbf{i} - 3\mathbf{j}$ and $\mathbf{c} = -5\mathbf{i} + 2\mathbf{j}$.

(a) Determine the angle between vectors $\mathbf{b}$ and $\mathbf{c}$, giving your answer rounded to one decimal place. (2 marks)

$$\cos^{-1}\frac{14\times(-5)+(-3)\times 2}{\sqrt{14^2+(-5)^2}\sqrt{(-3)^2+2^2}} = 170.3°$$

(Or using CAS)

(b) Find the position vector of point D which divides $\overline{AC}$ internally in the ratio $5:3$. (3 marks)

$$\mathbf{a} + \frac{5}{5+3}(\mathbf{c}-\mathbf{a}) = \begin{bmatrix}3\\4\end{bmatrix} + \frac{5}{8}\begin{bmatrix}-5-3\\2-4\end{bmatrix}$$

$$= \begin{bmatrix}3\\4\end{bmatrix} + \begin{bmatrix}-5\\-1.25\end{bmatrix}$$

$$= \begin{bmatrix}-2\\2.75\end{bmatrix}$$

$$\mathbf{d} = -2\mathbf{i} + 2.75\mathbf{j}$$

(c) Express the vector $\mathbf{b}$ in terms of $\mathbf{a}$ and $\mathbf{c}$. (4 marks)

$$\mathbf{b} = x\mathbf{a} + y\mathbf{c}$$

$$\begin{bmatrix}14\\-3\end{bmatrix} = x\begin{bmatrix}3\\4\end{bmatrix} + y\begin{bmatrix}-5\\2\end{bmatrix}$$

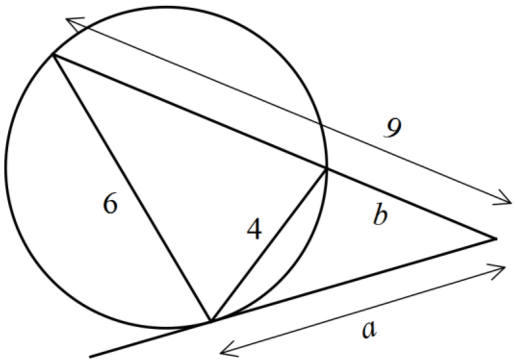
$$3x - 5y = 14$$
$$4x + 2y = -3$$

$$x = 0.5,\ y = -2.5$$

$$\mathbf{b} = 0.5\mathbf{a} - 2.5\mathbf{c}$$

Question 19 **(9 marks)**

(a) Determine the lengths a and b in the diagram below. (All dimensions in cm.) (3 marks)



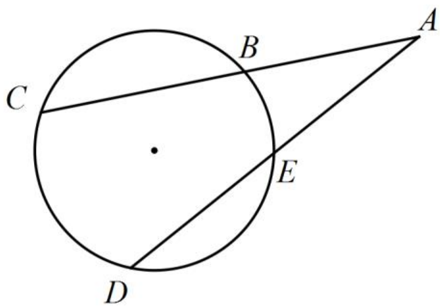
$\frac{4}{6} = \frac{a}{9}$ (Similar triangles)

$a = 6$

$6^2 = 9b$ (Tangent-secant theorem)

$b = 4$

(b) Two chords of a circle, CB and DE, are extended to meet at A.



(i) Prove that triangles ADC and ABE are similar. (3 marks)

$AB \times AC = AE \times AD$ (intersecting chords)

Hence

$\frac{AB}{AD} = \frac{AE}{AC}$ $\Rightarrow$ Two pairs of sides in equal ratio

$\angle A$ is common to both triangles

$\triangle ADC \sim \triangle ABE$

(ii) If $AB = 10$, $BC = 14$, $AE = 8$ and $BE = 4$ cm, determine the lengths of DE and CD. (3 marks)

$10 \times (10 + 14) = 8 \times (8 + DE)$

$30 = 8 + DE$

$DE = 22$ cm

$CD = \frac{30}{10} \times 4$

$= 12$ cm

Question 20 **(11 marks)**

Points A and B lie on level ground with position vectors $-21\mathbf{i}+10\mathbf{j}$ m and $-489\mathbf{i}+205\mathbf{j}$ m respectively. A steady wind with velocity $x\mathbf{i}+y\mathbf{j}$ is blowing across the ground.

In order to fly at a fixed height along the direct path from point A to point B, a small drone is programmed to fly with velocity $-30\mathbf{i}+16\mathbf{j}$ ms^{-1}.

(a) Show that the relationship between the coefficients x and y is given by $x=-2.4y-8.4$. (5 marks)

$$\overrightarrow{AB}=\begin{bmatrix}-489\\205\end{bmatrix}-\begin{bmatrix}-21\\10\end{bmatrix}=\begin{bmatrix}-468\\195\end{bmatrix}$$

$$t\left(\begin{bmatrix}-30\\16\end{bmatrix}+\begin{bmatrix}x\\y\end{bmatrix}\right)=\begin{bmatrix}-468\\195\end{bmatrix}$$

$$\frac{-468}{-30+x}=\frac{195}{16+y}\quad(=t)$$

$$\therefore x=-2.4y-8.4 \quad \text{(Using CAS)}$$

(b) Given that the wind has a speed of $\sqrt{37}$ ms^{-1}, calculate all possible wind velocities. (4 marks)

$$x^2+y^2=37$$

$$\left.\begin{cases}x=-2.4y-8.4\\x^2+y^2=37\end{cases}\right|_{x,y}\Rightarrow\{x=-6,y=-1\},\ \{x=3.515,y=-4.964\}$$

$$\mathbf{v}_w=\begin{bmatrix}-6\\-1\end{bmatrix}\text{ or }\begin{bmatrix}3.515\\-4.964\end{bmatrix}$$

(c) Determine all possible times for the drone to fly from A to B. (2 marks)

$$t=\left.\frac{195}{16+y}\right|_{y=-1}$$
$$=13 \text{ seconds}$$

$$t=\left.\frac{195}{16+y}\right|_{y=-4.964}$$
$$=17.7 \text{ seconds}$$

End of questions

Additional working space

Question number: _________

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